Bayesian estimation of a consumption system

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Why Bayesian approach?

- Handles problems with many parameters and little data.
- Allows for explicit formulation of prior knowledge.
- Shows how prior assumptions are updated by the data.
- Provides full account of uncertainty of the estimates.
- Single estimation strategy for a wide varety of models, including simultaneous equations.
- Allows straightforward handling of latent variables.
- Computationally feasible nowadays.

The cost compared to other approaches

- Requires full and explicit stochastic specification of the model.
  - Defining the likelihood function (sampling distribution).
  - Formulating prior knowledge (uncertainty) about model parameters in terms of probability distributions.

An alternative: Generalized Maximum Entropy

- Generalized Maximum Entropy / Generalized Cross Entropy (GME/GCE) approach introduced by Golan, Judge and Miller (1996).
- GME/GCE: "Robust estimation with limited data".
- Serious drawback: prior assumptions cannot be incorporated in a straightforward way.
- Heckelei, Mittelhemmer and Jansson (2008) demonstrated that seemingly uniform prior formulation actually imposes an informative prior; the authors proposed a Bayesian alternative.

Bayesian estimation: my previous attempts

- Matrix balancing – estimating a bridge matrix between NACE Rev. 1.1 and NACE Rev. 2 (published in CEJEME, 2016).
- Estimation of fixed capital stocks and depreciation rates (work in progress).
- Current work – at an early stage!

How it works

- Linear regression model: $y_t = \alpha x_t + \varepsilon_t$, where $\varepsilon_t \sim \mathit{Normal}(0, \sigma)$.
- Equivalent to: $y_t \sim \mathit{Normal}(\alpha x_t, \sigma)$.
- Likelihood function: $L(\alpha, \sigma; y) = \prod_t \mathit{normalpdf}(y_t | \alpha, \sigma)$
- Maximum likelihood: find α and σ that maximize L.
- Bayesian: we want to look at L for all possible values of α and σ.
  - Then e.g. calculate marginal distribution for α.
- Prior dsitribution provides additional weigthing.
- Posterior distribution: $p(\alpha, \sigma | y) \propto L(\alpha, \sigma; y) \cdot p(\alpha, \sigma)$.

Linear Expenditure System (LES)

- Also known as Stone-Geary or Klein-Rubin model.
- Restrictive assumptions but relatively few parameters.
- Still widely used, e.g. in the Computable General Equilibrium (CGE) field.
- Good grounds for testing the estimation method.

Linear Expenditure System formulation (1)

$$C_t \cdot w_{it}^* = \gamma_i \cdot p_{it} + \delta_i \left(C_t - \sum_j \gamma_j \cdot p_{jt} \right)$$

- w_{it}^*: predicted budget share of good i in total consumption.
- C_t: total nominal consumption expenditure.
- p_{it}: price of good i.

Estimated parameters:

- γ_i: 'subsistence' consumption of good i.
- δ_i: budget share of good i in 'non-subsistence' consumption (marginal budget share), where $\sum_i \delta_i = 1$.

Linear Expenditure System formulation (2)

- Parameters γ_i andδ_i can be used to derive income and price elasticities.
- Income (total expenditure) elasticity: $EC_i = \frac{\delta_{it}}{w_{it}^*}$
- Own price elasticity: $EP_{it} = \frac{1-\delta_i}{w_{it}^*} \cdot \frac{p_{it}\gamma_i}{C_t} - 1$

Stochastic specification

- Based on Osiewalski (2001):

$$w_{it} = w^*_{it}\left(p_t, C_t, \gamma, \delta\right) \cdot \varepsilon_{it}$$

- where w_{it} are the observed budget shares.
- Joint probability distribution is assigned to w_t vector directly.
- That distribution must satisfy: $\sum_i w_{it} = 1$.
- Common options: Dirichlet, Additive Logistic Normal (ALN).

The full model

$$w_{it}^* = \frac{1}{C_t} \left[\gamma_i \cdot p_{it} + \delta_i \left(C_t - \sum_j \gamma_j \cdot p_{jt} \right) \right]$$

$$w_t \sim ALN\left(w_t^*, \Sigma\right)$$

$$\Sigma \sim SomeNonInformativePrior\left(\ldots\right)$$

$$\gamma_i \sim Uniform\left(\ldots\right)$$

$$\delta \sim Dirichlet\left([1, \ldots, 1]\right)$$

- The above formulation is slightly stylized, but model coding follows it closely.

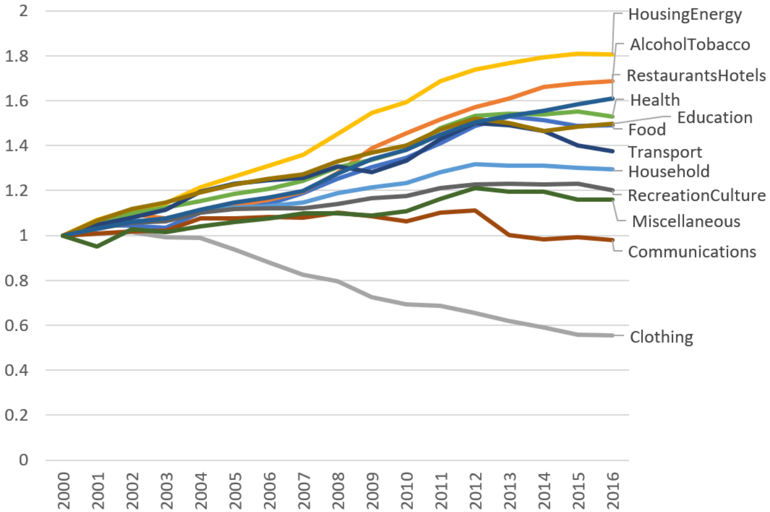
Software: Stan

- Programming language for Bayesian inference with MCMC sampling.
- Open source, active user forum.
- Interfaced with R, Python, MATLAB, Stata, etc.
- Available at mc-stan.org

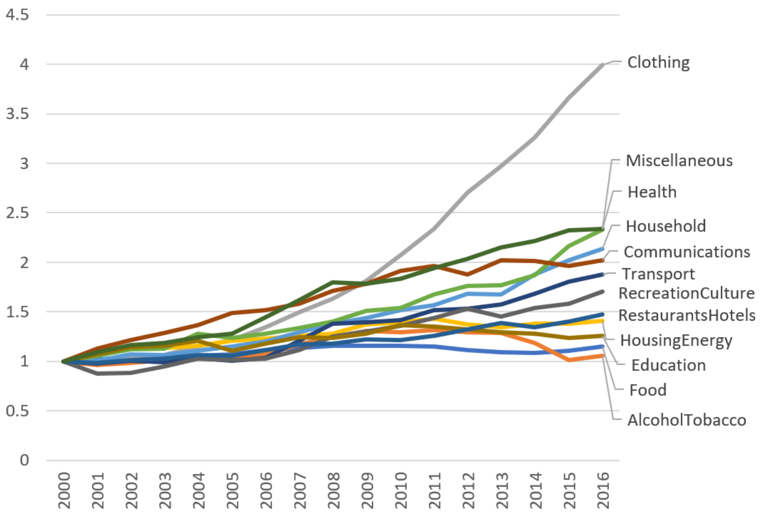
Data for Poland

- Eurostat: consumption expenditure by COICOP.
- 12 COICOP groups, annual, 2000-2016.
- Data categories used in the estimation:
  - Consumption in current prices.
  - Consumption deflators.

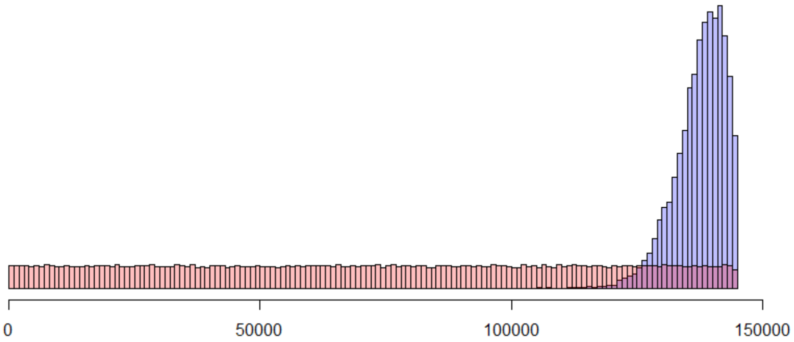
Prices of consumption goods, 2000=1



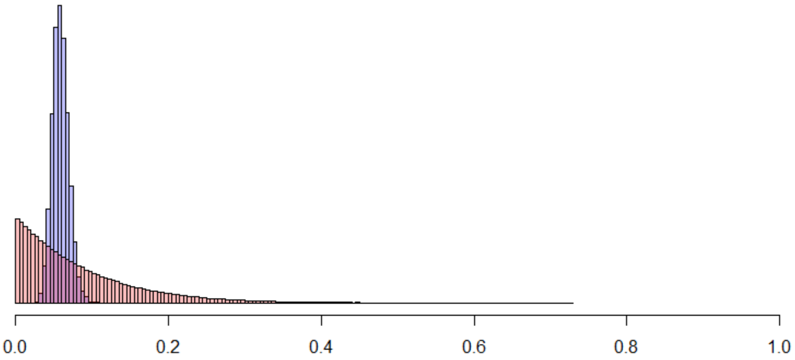
Consumption in constant prices, 2000=1



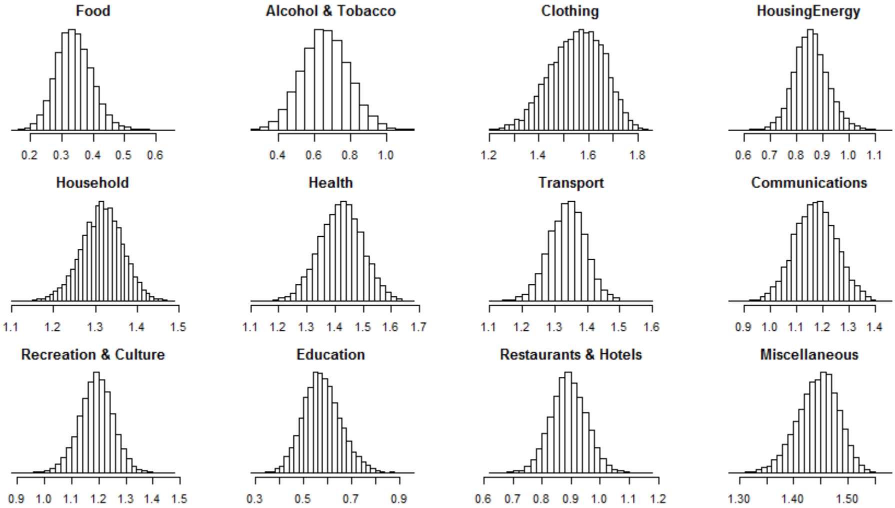
Subsistence consumption of food (γ_1): samples from prior an posterior distributions



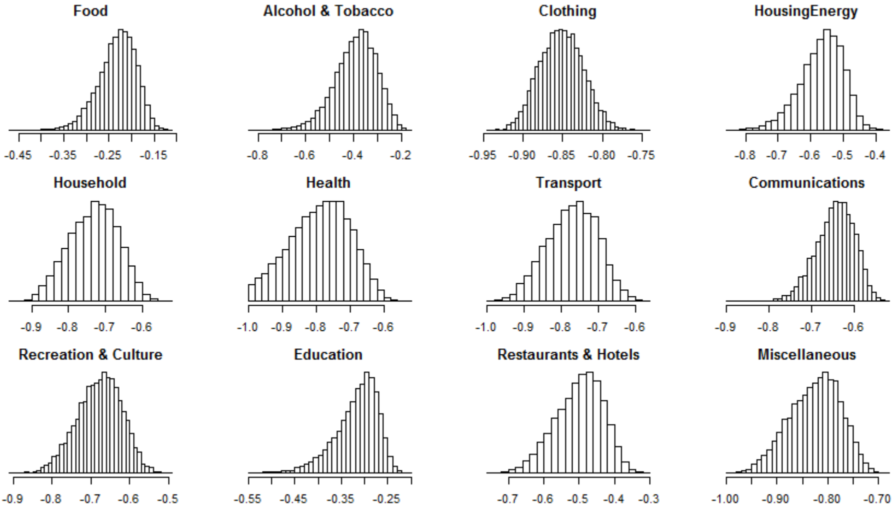
Marginal budget share of food consumption (δ_1): samples from prior an posterior distributions



Income elasticities (at 2016 income and price levels)



Own price elasticities (at 2016 income and price levels)



Posterior means of the elasticities

Good	*Income elasticity*	*Price elasticity*
Food	0.34	−0.23
Alcohol & Tobacco	0.66	−0.39
Clothing	1.55	−0.85
HousingEnergy	0.85	−0.57
Household	1.32	−0.73
Health	1.42	−0.79
Transport	1.34	−0.77
Communications	1.17	−0.65
Recreation & Culture	1.19	−0.68
Education	0.58	−0.32
Restaurants & Hotels	0.89	−0.50
Miscellaneous	1.44	−0.83

Further steps

- LES: allowing for a time-varying subsistence consumption.
- Switching to more flexible demand systems, e.g. PADS.