



Thoughts on Input-Output Models in National Accounting Systems
With “Superlative” and Chain-Weighted Indexes

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with provocation by
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Beginning of the Trouble

“You can see the computer age everywhere but in the productivity statistics.”
Robert Solow, *New York Times Book Review*, 1987

Solow was wrong, but the Bureau of Economic Analysis (BEA), supposed that he was right and set about to make a hedonic index of computer prices that was used to deflate investment, export, and personal consumption of computers in GDP.

The price of computers proved to be falling at about 15 percent per year, while the current-price value was about constant, so the “real” growth in the computer component of GDP began to rise at about 15 percent per year.

BEA could have scraped the hedonic computer deflator, but instead turned to index-number theorists and introduced the use of Fisher’s “ideal” index between pairs of adjacent years and chaining of indexes over periods of more than two years.

The “Ideal” Indexes

The Quantity index between year 1 and 2 is

$$Qq(2,1) = \sqrt{((p1 \cdot q2)/(p1 \cdot q1) * (p2 \cdot q2)/(p2 \cdot q1))}$$

The Price index between the two periods is

$$Pq(2,1) = \sqrt{((p2 \cdot q1)/(p1 \cdot q1) * (p2 \cdot q2)/(p1 \cdot q2))}$$

Chaining

The quantity index between year 1 and some subsequent year, say 4, is

$$Qq(4,1) = Qq(2,1) * Qq(3,2) * Qq(4,3)$$

The price index is

$$Pq(4,1) = Pq(2,1) * Pq(3,2) * Pq(4,3)$$

Values in Chained Constant Prices

The value of vector q in year 4 “in chained prices of year 1 is then:

$$Vq(4,1) = (p1 \cdot q1) * Qq(4,1).$$

So What is the Problem?

If $q = x + y$ only by rare accident will we have

$$Vq(t,1) = Vx(t,1) + Vy(t,1) \text{ for } t \neq 1.$$

So, in years other than the base of the prices,

$$\text{gdp}_R \neq c_R + v_R + g_R + x_R - m_R$$

$$c_R \neq cd_R + cn_R + cs_R$$

In a 400-sector input-output table made from information on 10,000 products, gross product of a sector in constant chained prices is NOT equal to the sum of the constant-price values of the cells in the row!

The situation cannot be fixed by chain weighting ourselves, unless we are working at the level of several thousand products used by BEA. But BEA will never again release data at that level.

Cross-Weighted Terms Cannot be Deduced

From $(p_1 \cdot q_2) \cdot (p_2 \cdot q_2) / (p_1 \cdot q_1)(p_2 \cdot q_1) = Q$

we deduce that

$$(p_1 \cdot q_2) / (p_2 \cdot q_1) = Q \cdot (p_1 \cdot q_1) / (p_2 \cdot q_2)$$

While from the price index, we get

$$(p_1 \cdot q_2) / (p_2 \cdot q_1) = (p_2 \cdot q_2) / P \cdot (p_1 \cdot q_1)$$

Thus, we cannot deduce $p_1 \cdot q_2$ and $p_2 \cdot q_1$.

So we cannot calculate aggregates of official quantities and get what would be the official aggregates.

What should we do?

We must make up our own tables and national accounts in constant prices.

How? Pick price indexes for each sector. Use them to put current price tables into constant prices. Add up final demands columns to get GDP components and GDP.

The price indexes can be scaled to give the official GDP if desired.

