

# Consumption of Motor Vehicles based on Capital Stock Data

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# The Task

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- Create new times series of gross and net stocks of motor vehicles in the households in Denmark. Also scrappage and depreciation must be measured as well as the user cost of capital.
- Use the new data on user cost and capital to replace current measures of the “consumption” of vehicles as opposed to purchase.
- Implement net capital stocks as a representation of total value of vehicles in wealth measures.
- Use these new series to estimates a model of purchase of vehicles in private consumption in ADAM as a replacement for the current model.

# Definitions

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- Consumption of motor vehicles  $\approx$  consumption of cars
- Gross Capital Stock – capital stock measured in today's prices as if all goods were new – a volume measure – a measure of productive capacity.  $fKcb$
- Net Capital Stock - capital stock measured in second hand market prices reflecting their actual value given their age and condition. A measure of value.  $fKncb$
- User cost of capital. The cost of owing one unit of capital in a given period. The price of using the capital.  $Ucb$
- Consumption of capital is  $Ucbs = Ucb * fKcb(-1)$

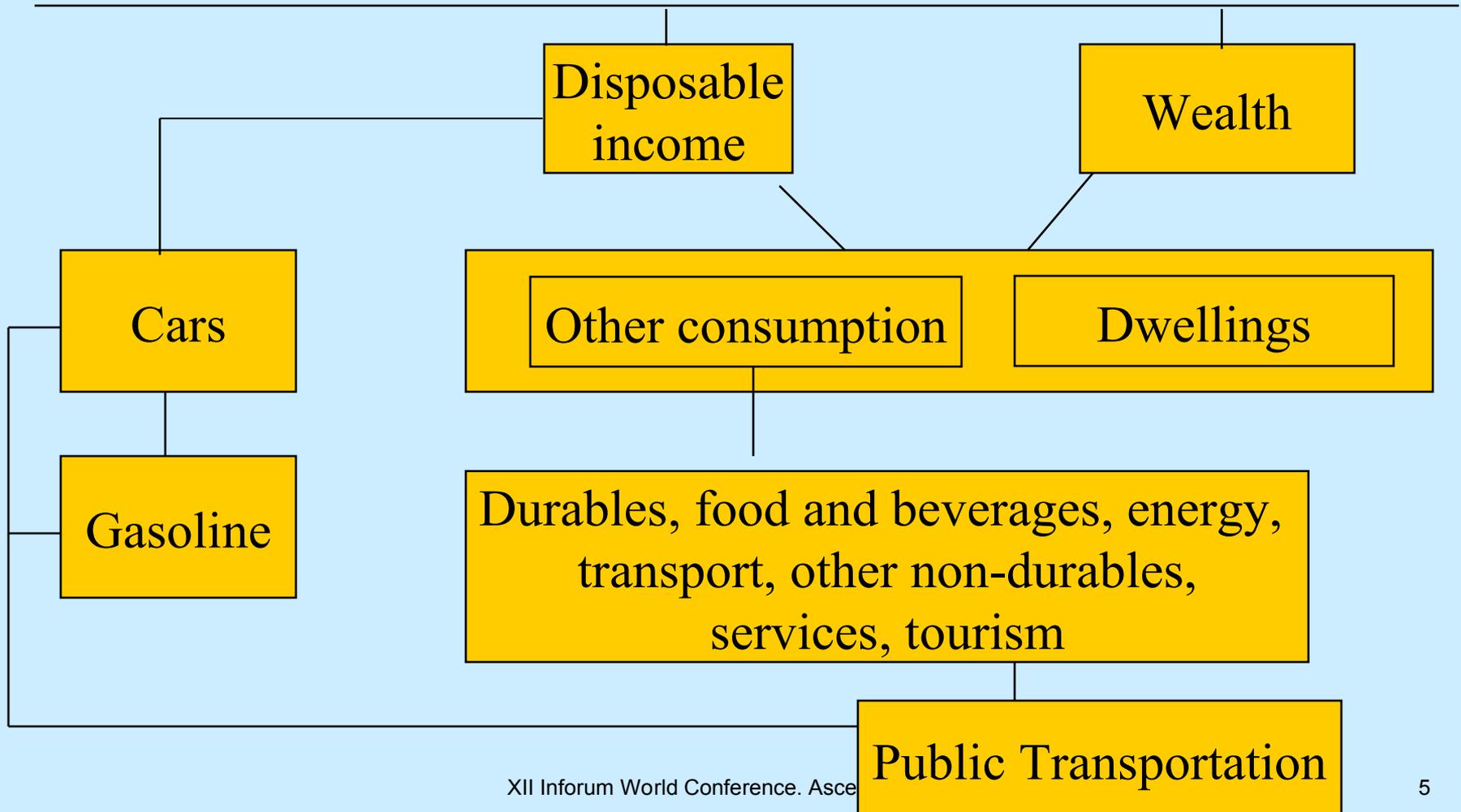


# Background

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- Current model is 20 years old and involves a number of shortcuts that are not necessary to make today, due to better data.
- Net capital stock of vehicles that constitutes a good part of the wealth of households was about 1/6 of the expected actual value due to very high depreciation rates (not updated for 20 years).
- Better statistics on the lives of single vintages of vehicles.

# The consumption system in ADAM



# Background

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- Previous car purchase model based on the “Capital adaptation principle” which is the following

$$\Delta K = \beta (K^* - K_{-1}) \quad \text{or} \quad \Delta K = \beta (K_{-1}^* - K_{-1})$$

- However, the equation was rewritten with a Stone Rowe Transformation, because capital stock data was not available previously. With this it is possible to express change in investment  $\Delta I$  only in terms of explanatory variables like income and price.
- The idea now was to get capital stock data and then to skip the transformation



# Capital Stock Estimation

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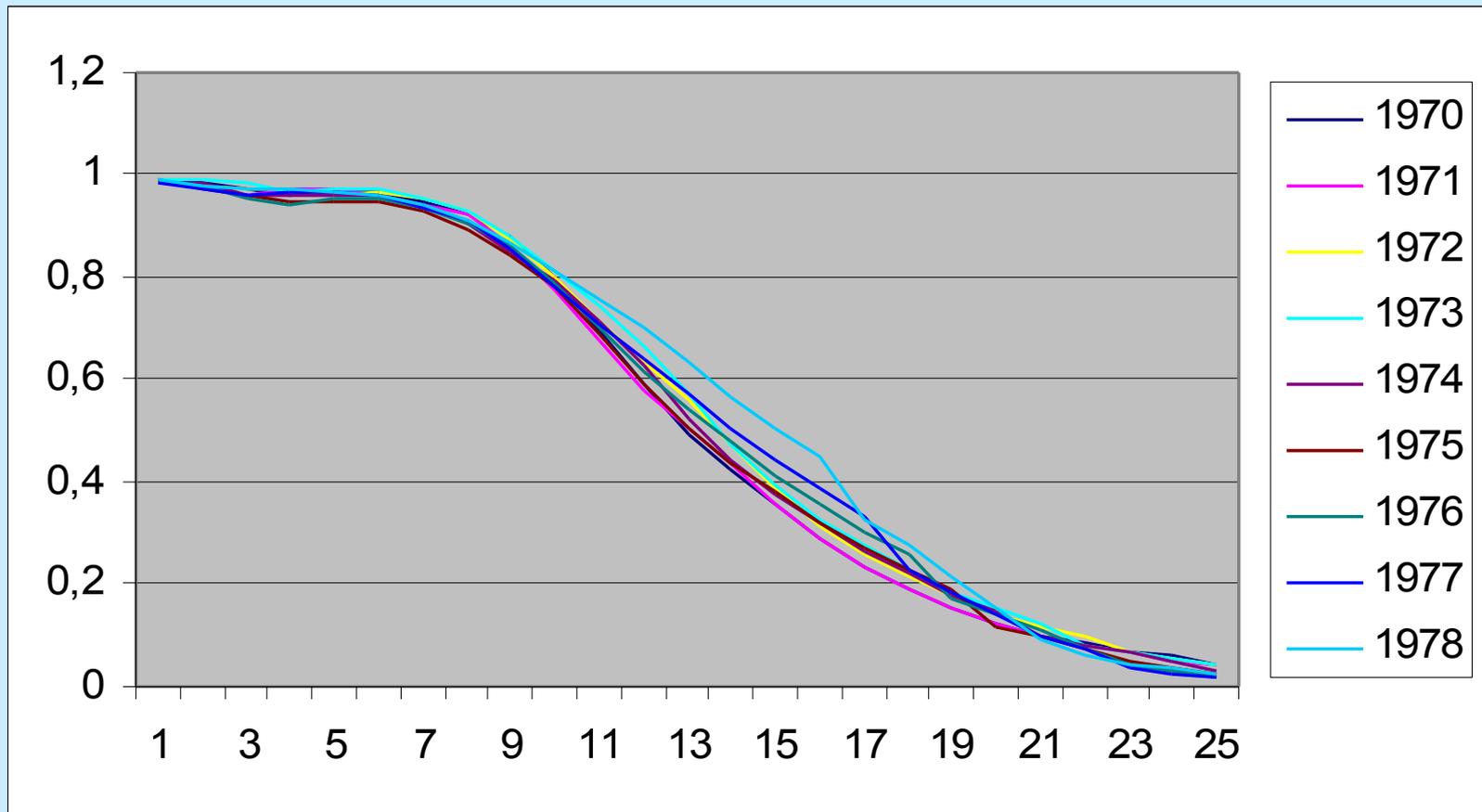
- First set out to use the Perpetual Inventory Method (PIM). Basically it keeps track of all vintages of capital that are supposed to decline along a survival curve.
- There is an amount of different survival curves available like Winfrey curves, exponential curves, curves based on a normally distributed scrapping curve, Weibull and so on.
- If one has an estimate of average life time for the good in question, then just pick a curve and feed in investment and the capital stock will soon be ready
- Otherwise the curve and average life time can be estimated like in the this case

# National car register

1970	107506	106847	105566	104003	105109	105270
1971		102411	101566	99707	100795	100960
1972			90875	90421	90062	90247
1973				120371	120347	119432
1974					77673	77755
1975						114741
1976						
1977						
1978						
1979						

Number of cars from a particular vintage still in use

# Actual survival curves



# Use of Curves

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- Fit a functional form to the curves. Here the Weibull form

$$B_s(x) = 1 - \int_{y < x} \alpha \lambda \cdot (\lambda x)^{\alpha - 1} \cdot e^{-(\lambda x)^\alpha} dy = e^{-(\lambda x)^\alpha}$$

- Lambda is a size parameter, alpha is a form parameter

$$b_s(x) = \alpha \lambda \cdot (\lambda x)^{\alpha - 1} \cdot e^{-(\lambda x)^\alpha} \quad \text{for } x \geq 0$$

- $b_s(x)$  = scrapping curve – the derivative of  $B_s(x)$ . It is bell-shaped. The 2 parameters estimated for the 1970-1977 curves with the following results

# Estimation results

Årgang	Alpha	Lambda	R-sq	E(X)
1970	3,4915	0,0682	0,84	13,19
1971	3,3864	0,0677	0,86	13,27
1972	3,4510	0,0657	0,89	13,69
1973	3,5098	0,0653	0,91	13,78
1974	3,3126	0,0662	0,85	13,56
1975	3,0910	0,0657	0,84	13,61
1976	2,9680	0,0631	0,78	14,15
1977	2,9200	0,0613	0,83	14,56
1978	3,1060	0,0577	0,86	15,49
1979	3,3560	0,0555	0,84	16,16

# A graph of the 1977 result



# Time series of survival curves

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- The average service life  $E(x)$  can be estimated by the use of the parameters from the Weibull:

$$E(X) = \frac{1}{\lambda} \cdot \Gamma\left(1 + \frac{1}{\alpha}\right)$$

- By assumptions about the development of average service life from 1950 to 2000 and turning this formula around it was possible to create a time series of survival functions to go into

$$fKcb_t = \sum_{s=0}^{\infty} fKcb_{t,s} = \sum_{s=0}^{\infty} B_s \cdot fCb_{t-s}$$

- $fCb$  is "investment" i.e. purchase of vehicles (1948-2003)



# A new development

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- At some point it was revealed that the National Accounts would publish gross and net stocks and depreciation for vehicles in household consumption 1993-2001. Based on register data only and with second hand prices 93-2001 for 25 vintages of a large number of different brands and models.
- Now the task changed to create consistent series 1950-1993 that would equal the new series exactly in 1993!!
- The previously estimated series did NOT fit. Only by raising average service life by almost 2 years back in time would make them fit. There had to be another way..

# Backwards in time

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- The dynamic equality between capital stock and investment

$$K_t = K_{t-1} + I_t - \delta K_{t-1} \Leftrightarrow$$

$$I_t = K_t - (1 - \delta) K_{t-1}$$

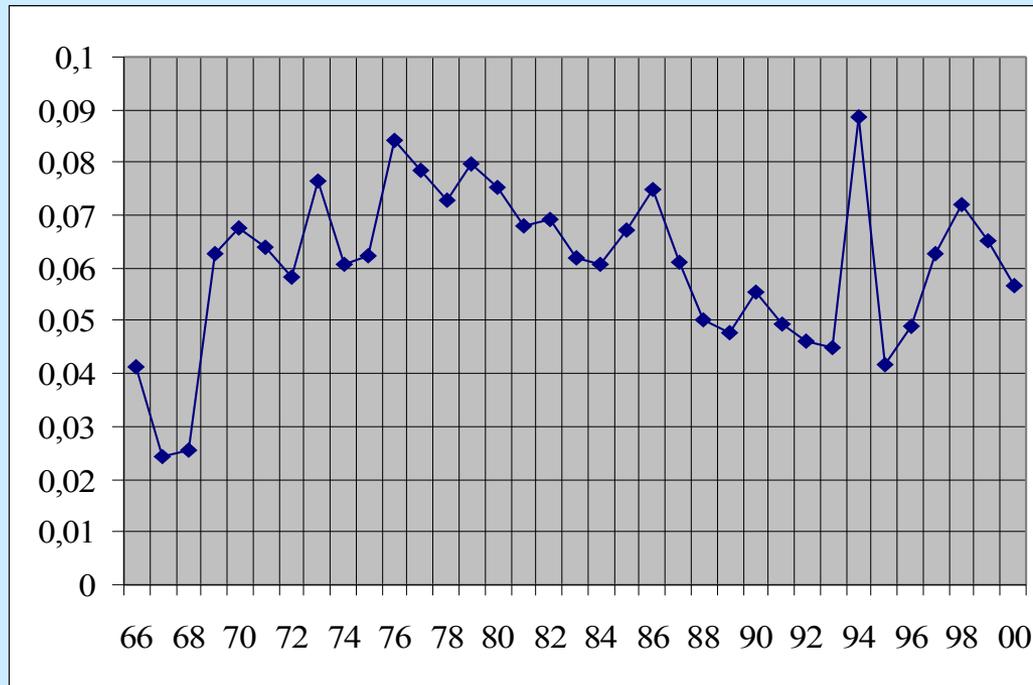
- This equation can be rewritten so we have  $K_{t-1}$  on the l.h.s.

$$K_t = \frac{K_{t+1} - I_{t+1}}{(1 - \delta_{t+1})}$$

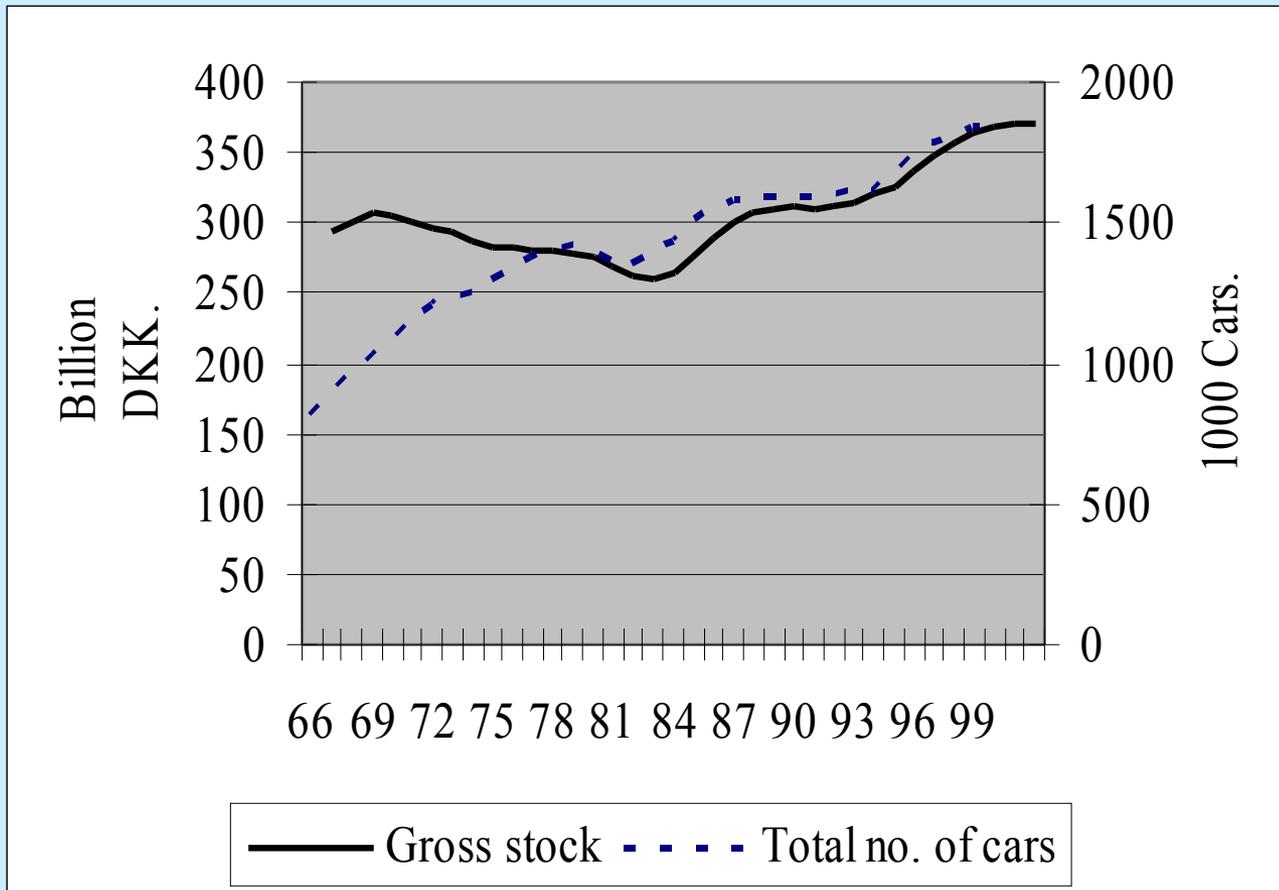
- We know  $I_{t+1}$  (fCb) and  $K_{t+1}$  so it only requires knowledge about the scrapping  $\delta_{t+1}$ . Could come from the national car register.

# Physical scrapping rate

$$\delta_t = \frac{\text{Stock in numbers}_{t-1} - \text{Stock in numbers}_t + \text{New entries to stock}_t}{\text{Stock in numbers}_{t-1}}$$

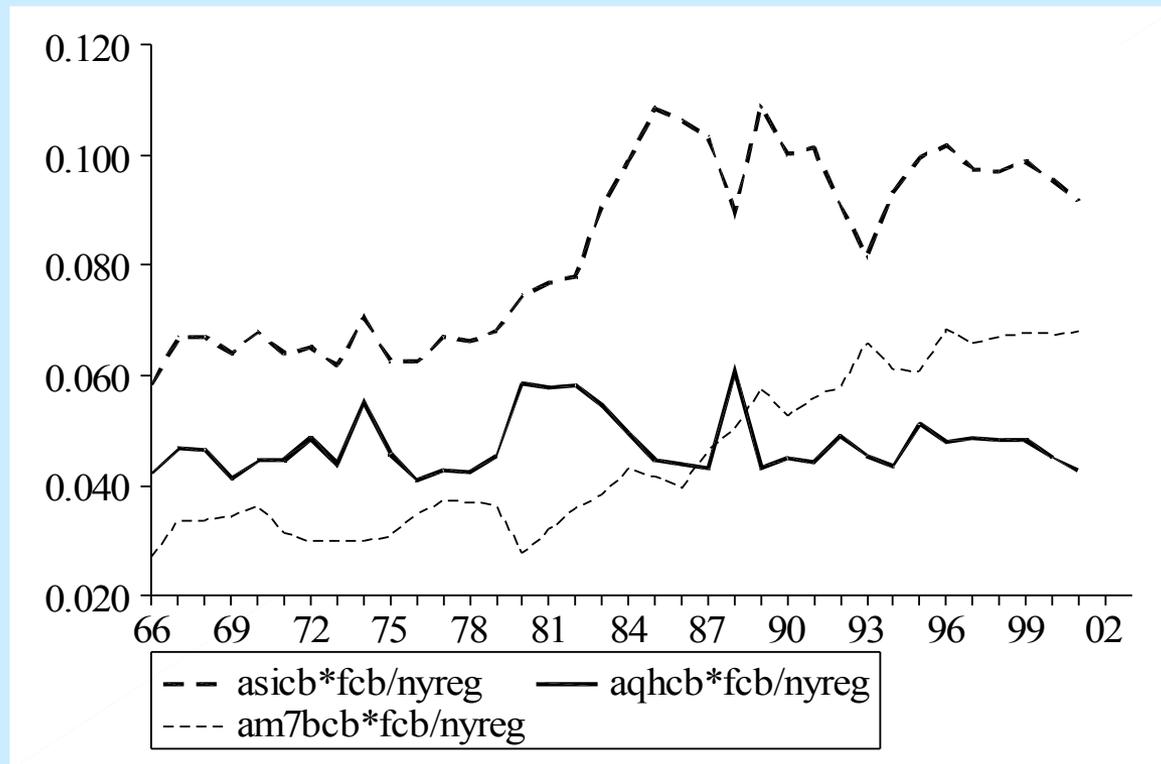


# ”Backwards” estimate of gross stock

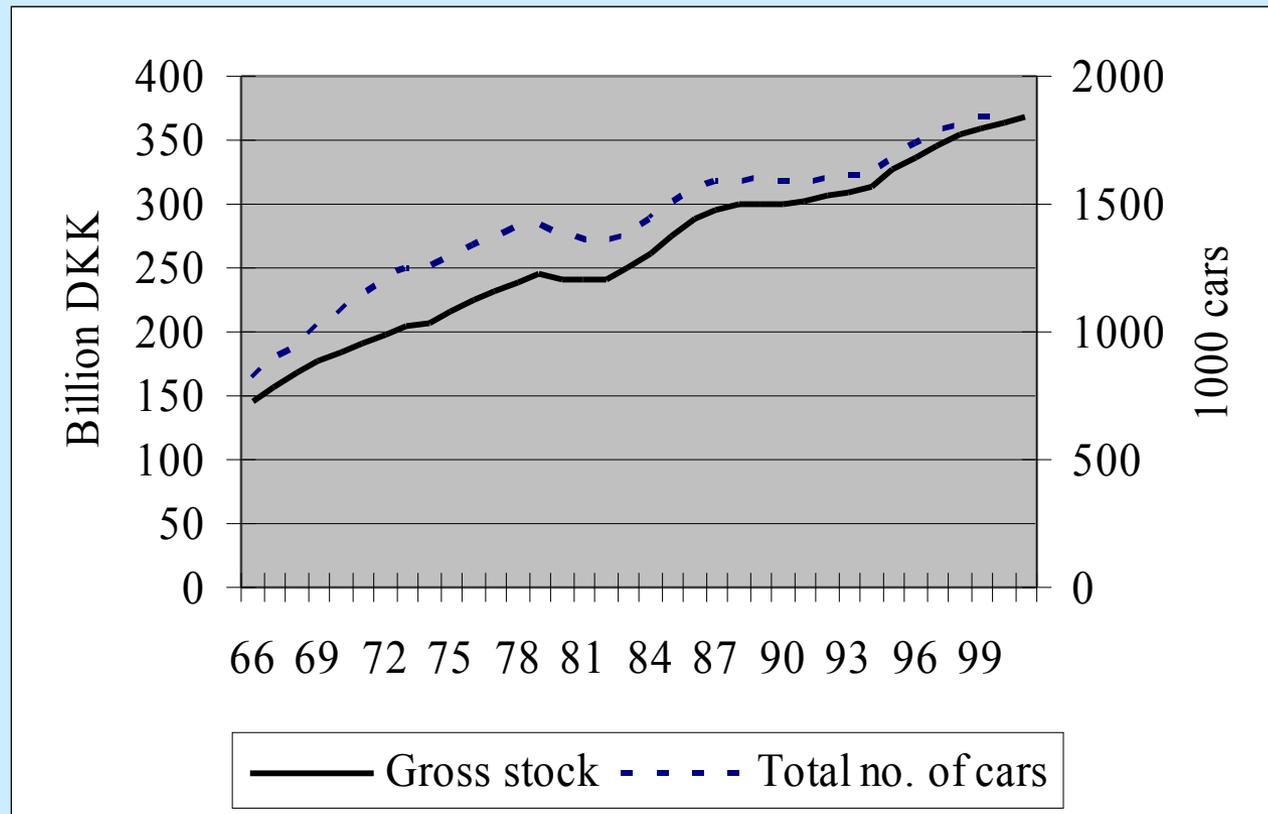


# A trend in fixed price value of cars

- Consumption of vehicles in fixed 1995 prices relative to no. of new cars sold was not constant.



# Final estimate of gross stock



# Net stock and depreciation

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- Normally, the net stock would be made by estimation of depreciation and then subtraction of that from the gross capital.
- That also happens in the Danish NA. Depreciation is roughly  $1/L$  of the remaining gross stock, with  $L$  being expected service life. However, the depreciation formula used by the NA could NOT give the level of net stock in 1993.
- A formula suggested by BEA did the job. Modified double declining balance with  $\kappa=1.85$

$$fKncb_t = \left( 1 - \frac{\kappa}{L} \right) \cdot fKncb_{t-1}$$

- Note that the gross stock is not necessary at all here.

# User cost of capital

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- Could be regarded as the rental price for the service flow provided by the capital stock at time  $t$ .

$$ucb = \frac{fkncb_{-1}}{fkcb_{-1}} \cdot pcb \cdot \left( i(1-t) + \frac{finvcb}{fkncb_{-1}} - rpcbe \right) + \frac{sdv}{fkcb_{-1}}$$

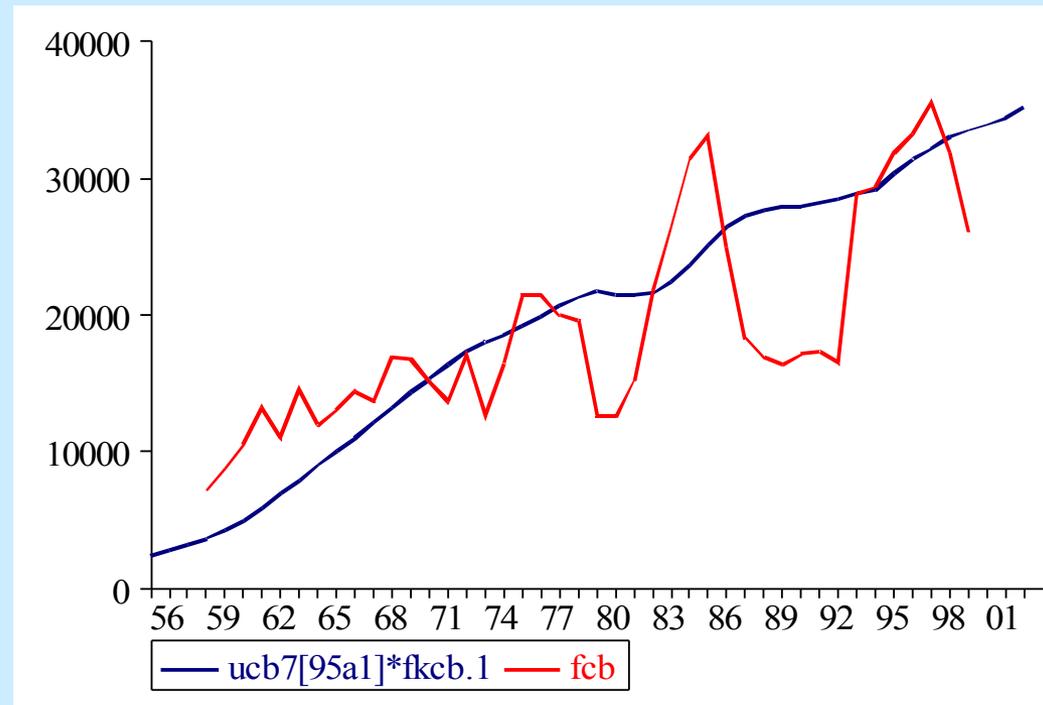
where

$$rpcbe = 0.4 \cdot \frac{pcb - pcb_{-1}}{pcb_{-1}} + (1 - 0.4) \cdot rpcbe_{-1}$$

- Where  $pcb$  is the price for new cars and  $sdv$  is car tax.  $Rpcbe$  is the expected capital gain. The first term is a little strange but has a long tradition in ADAM.

# Consumption of motor vehicles

- With the stock and the user cost ready, we can calculate the consumption as the annual “flow” from the stock as  $ucb*fkcb(-1)$  (current prices) and  $ucb[95a1]*fkcb(-1)$  (fixed prices).





# Long run desired level of $fKcb$

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- We need to estimate a  $K^*$  for the capital equation.

# Equations

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□ **Long run desired level**

□  $fKcb = a_0 + a_1 * (income) + a_2 * (wealth.1) + a_3 * (ucb) ;$

□ **With capital adaptation**

□  $fKcbw = a_0 + a_1 * (income) + a_2 * (wealth.1) + a_3 * (ucb) ;$

□  $dfkcb = \gamma_0 * (fkcbw(-1) - fkcb(-1)) ;$

□ **Real error correction model**

□  $fKcbw = a_1 * (income) + a_2 * (wealth) + a_3 * (ucb) + a_4 * (gasPrice) ;$

□  $dfkcb = \gamma_0 * (fkcbw(-1) - fkcb(-1)) +$

□  $\gamma_1 * (fkcbw - fkcbw(-1)) ;$

□ **Error correction with free effects in short run**

□  $fc_b = \gamma_0 * (fkcbw(-1) - fkcb(-1)) +$

□  $c_1 * dincome + c_2 * dwealth.1 + c_3 * ducb + c_4 * (dgasPrice) +$

□  $bfivcb * fkcbw(-1) ;$

□ (because  $dfkcb = fcb - bfivcb * fkcbw(-1)$ )

# Results, long run equation

Parameter	Estimate	Error	t-statistic	P-value
Constant	-135655.	13259.2	-10.2310	[.000]
DispIncome	.354909	.056226	6.31215	[.000]
Wealth	.127504	.649794E-02	19.6223	[.000]
Usercost	-72530.7	32943.0	-2.20170	[.028]

Mean of dep. var.	=	214137.
Std. dev. of dep. var.	=	98293.6
Sum of squared residuals	=	.290123E+10
Variance of residuals	=	.725307E+08
Std. error of regression	=	8516.50
R-squared	=	.993017
Adjusted R-squared	=	.992493
LM het. test	=	.286178 [.593]
Durbin-Watson	=	.605240 [.000, .000]

# Result, with capital adaptation

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Standard	Parameter	Estimate	Error	t-statistic	P-value
	Const	-113424.	29335.9	-3.86637	[.000]
	Dispinc	.699705	.161966	4.32008	[.000]
	Wealth	.072645	.022701	3.20010	[.001]
	Usercost	-101607.	55987.3	-1.81482	[.070]
	GAMMA0	.184557	.059966	3.07768	[.002]

Mean of dep. var. = 7601.23  
Std. dev. of dep. var. = 4228.35  
Sum of squared residuals = .433733E+09  
Variance of residuals = .111214E+08  
Std. error of regression = 3334.87  
R-squared = .435828  
Adjusted R-squared = .377964  
LM het. test = 1.35890 [.244]  
Durbin-Watson = .858891 [.000, .000]

# Result with error correction

Parameter	Estimate	Error	t-statistic	P-value
DispInc	.224433	.047978	4.67779	[.000]
Wealth	.125601	.720394E-02	17.4351	[.000]
Usercost	-211445.	46041.8	-4.59245	[.000]
GasolinPr	-42018.7	5195.99	-8.08676	[.000]
GAMMA0	.269640	.041487	6.49945	[.000]
GAMMA1	.346836	.046588	7.44480	[.000]

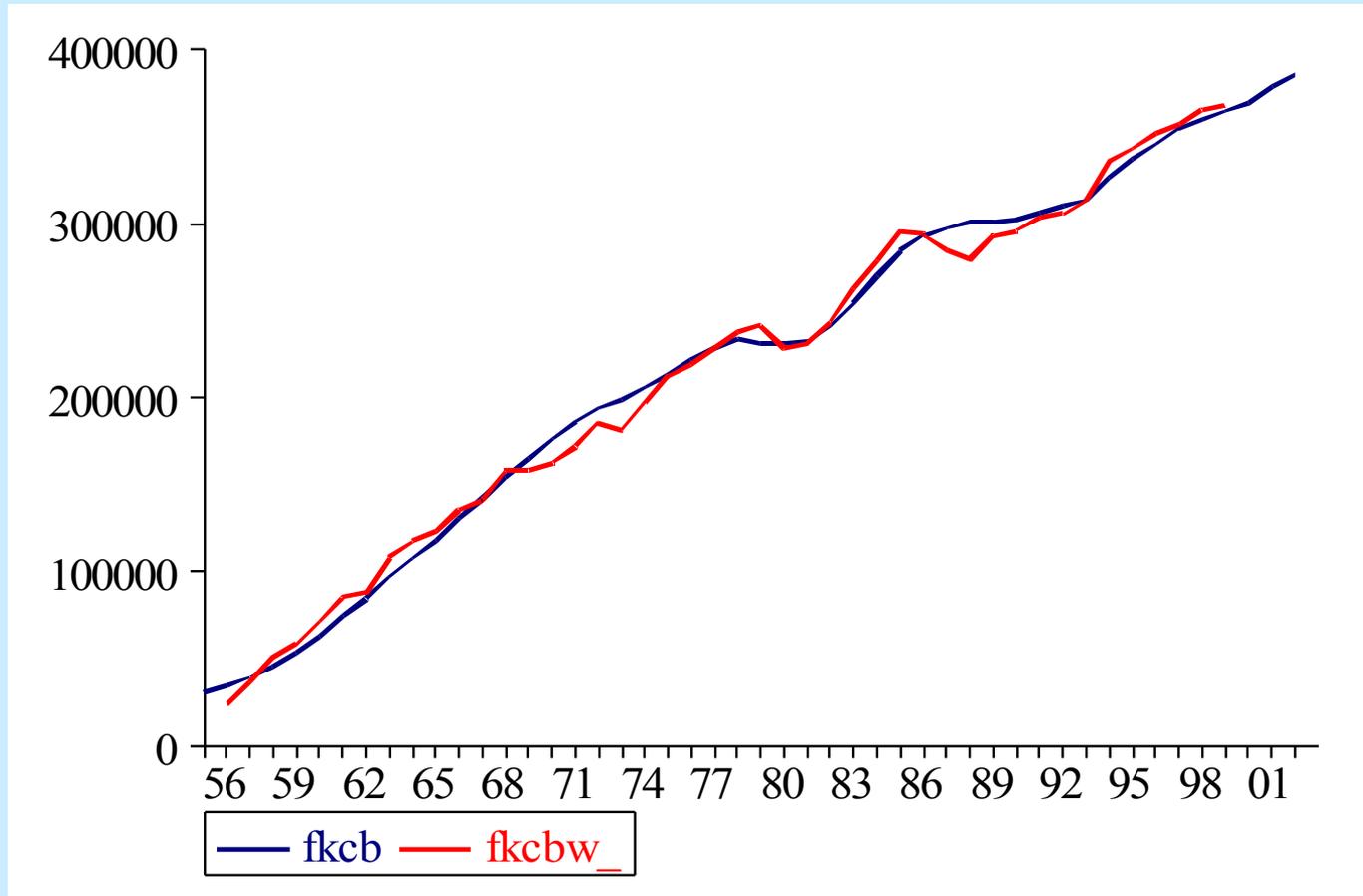
- Mean of dep. var. = 7666.44
- Std. dev. of dep. var. = 4255.95
- Sum of squared residuals = .215103E+09
- Variance of residuals = .581359E+07
- Std. error of regression = 2411.14
- R-squared = .717281
- Adjusted R-squared = .679076
- LM het. test = 5.39588 [.020]
- Durbin-Watson = 1.06603 [.000, .008]

# Error correction, free short run

□	Standard				
□	Parameter	Estimate	Error	t-statistic	P-value
□	Income	.290187	.066568	4.35928	[.000]
□	Wealth	.115513	.010207	11.3169	[.000]
□	User cost	-257077.	63004.1	-4.08033	[.000]
□	GasPrice	-41438.9	5894.77	-7.02977	[.000]
□	GAMMA0	.223070	.050623	4.40653	[.000]
□	Income	.037057	.032584	1.13729	[.255]
□	Wealth	.038885	.882263E-02	4.40739	[.000]
□	User cost	-112872.	66444.0	-1.69875	[.089]
□	GasPrice	-19298.1	5007.75	-3.85364	[.000]

- Mean of dep. var. = 18872.3
- Std. dev. of dep. var. = 7739.86
- Sum of squared residuals = .201606E+09
- Variance of residuals = .592960E+07
- Std. error of regression = 2435.08
- R-squared = .919880
- Adjusted R-squared = .901028
- LM het. test = .060828 [.805]
- Durbin-Watson = 1.02295 [.000, .023]

# Results, long run model plus error correction models.



# Results, AR1 construction

Parameter	Estimate	t-statistic	P-value
Konst	-103714.	-4.96490	[.000]
Inco_L	.611472	5.80403	[.000]
Weal_L	.085671	6.04604	[.000]
Ucb_L	-418357.	-2.26170	[.024]
GAMMA0	.217920	4.02768	[.000]
Inco_S	.086185	3.76684	[.000]
Weal_S	.028256	4.00519	[.000]
Ucb_S	-132963.	-3.21316	[.001]
Gasp_S	-4765.22	-1.56249	[.118]
D80_S	-12999.7	-1.69502	[.090]
RHO	.710431	5.93047	[.000]

Mean of dep. var. = 19203.6
Std. dev. of dep. var. = 7518.68
Sum of squared residuals = .808716E+08
Variance of residuals = .260876E+07
Std. error of regression = 1615.17
R-squared = .965271
Adjusted R-squared = .954068
LM het. test = .710241 [.399]
Durbin-Watson = 2.37482 [.226,1.00]

# Results, lagged endogenous variable

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Parameter	Estimate	t-statistic	P-value
Const	-124789.	-6.92792	[.000]
Inco_L	.642169	6.45433	[.000]
Weal_L	.086646	6.84314	[.000]
Ucb_L	-341231.	-3.22429	[.001]
GAMMA0N	.120257	3.21926	[.001]
GAMMA9N	.489485	5.40058	[.000]
Inco_S	.078836	3.12471	[.002]
Weal_S	.021110	3.10695	[.002]
Ucb_S	-141008.	-2.95428	[.003]
Gasp_S	-10536.3	-2.87977	[.004]

Mean of dep. var. = 18872.3  
Std. dev. of dep. var. = 7739.86  
Sum of squared residuals = .106325E+09  
Variance of residuals = .322197E+07  
Std. error of regression = 1794.99  
R-squared = .958248  
Adjusted R-squared = .946861  
LM het. test = .026782 [.870]  
Durbin-Watson = 2.35820 [.299, .999]

# Results, AR1 and lagged endogenous models (quite similar results)

